Pochodne

Zad 1. Oblicz pochodną funkcji:

(a) $f(x)=\sqrt[3]{x}-\sqrt{2}$; **(b)** $f(x)=\frac{2x^3-3x+\sqrt{x}-1}{x}$; **(c)** $f(x)=\frac{2}{\sqrt[3]{x^2}}-\sqrt[3]{x}$; **(d)** $f(x)=7x\sin x$;
(e) $f(x)=\sqrt{x}\operatorname{arc\,tg} x$; **(f)** $f(x)=x^3\ln x$; **(g)** $f(x)=\frac{1}{2}x^2\sin x\log x$; **(h)** $f(x)=\frac{1}{\log_2 x}$;
(i) $f(x)=\frac{\sin x}{1+\cos x}$; **(j)** $f(x)=\frac{5x^2+x-2}{x^2+7}$; **(k)** $f(x)=\frac{\cos x}{e^x}$; **(l)** $f(x)=\frac{x}{1+x^2}-\arctan x$;
(m) $f(x)=\frac{\sqrt[3]{x}}{1-\sqrt[3]{x}}$; **(n)** $f(x)=\frac{\arccos x}{x}$; **(o)** $f(x)=\frac{10^x}{e^x\sin x}$; **(p)** $f(x)=\frac{4^x\sin x}{1+2x^3\operatorname{tg} x}$;

Zad 2. Oblicz pochodną funkcji:

(a) $f(x)=(1+x^2)^6$; **(b)** $f(x)=(3x+x^2)^3$ **(c)** $f(x)=\cos 2x$; **(d)** $f(x)=\sin(1+7x)$;
(e) $f(x)=\operatorname{tg}(\frac{1}{3}x)$; **(f)** $f(x)=\sqrt{1+x^2}$; **(g)** $f(x)=\sqrt{\sin x+2x^2}$; **(h)** $f(x)=\sqrt{\frac{1+x}{1-x}}$;
(i) $f(x)=\sqrt[3]{(x^2+x-2)^2}$; **(j)** $f(x)=\operatorname{tg}^4 x$; **(k)** $f(x)=\cos^2 x$; **(l)** $f(x)=\sqrt{\operatorname{tg}(\frac{x}{2})}$;
(m) $f(x)=\arcsin\frac{2x-1}{\sqrt{3}}$; **(n)** $f(x)=\sqrt{\ln x}$; **(o)** $f(x)=\sin\sqrt{1+x^2}$; **(p)** $f(x)=\sin(\sin x)$;
(q) $f(x)=\sqrt{2+\operatorname{tg}\left(x+\frac{1}{x}\right)}$; **(r)** $f(x)=\cos^2\frac{1-\sqrt{x}}{1+\sqrt{x}}$; **(s)** $f(x)=e^{\sqrt{\ln x}}$; **(t)** $f(x)=2^{3^x}$;
(u) $f(x)=\sin\left(e^{x^2+3x+2}\right)$; **(v)** $f(x)=\ln(\sin 8x)$; **(w)** $f(x)=\log^4\sqrt{1+x^4}$; **(x)** $f(x)=e^{\frac{x}{\ln x}}$;
(y) $f(x)=\frac{x\sin(1+x^2)}{\sqrt{1+x^3}}$; **(z)** $f(x)=\log_2(\log_3(\log_5 x))$;

Zad 3. Oblicz pochodną funkcji:

(a) $f(x)=x^x$; **(b)** $f(x)=x^{x^2}$; **(c)** $f(x)=(\sin x)^{\cos x}$; **(d)** $f(x)=x^{\ln x}$; **(e)** $f(x)=(x+1)^{\frac{2}{x}}$;
(f) $f(x)=\sqrt[x]{x}$; **(g)** $f(x)=x^{\sqrt{x}}$; **(h)** $f(x)=x^{e^x}$; **(i)** $f(x)=\left(1+\frac{1}{x}\right)^x$; **(j)** $f(x)=(\ln x)^x$;
(k) $f(x)=x^{x^x}$; **(l)** $f(x)=(\ln x)^{e^x}$; **(m)** $f(x)=(\operatorname{tg} 2x)^{\operatorname{ctg}\frac{x}{2}}$;

Zad 4. Obliczyć $f'(x), f''(x), f'''(x)$ dla funkcji:

(a) $f(x)=x^3-\frac{2}{x}$; **(b)** $f(x)=x\sin x$; **(c)** $f(x)=\frac{e^x}{x}$; **(d)** $f(x)=x^4\ln x$; **(e)** $f(x)=e^{\cos x}$;

Zad 5. Funkcja g ma pochodne do drugiego rzędu włącznie. Obliczyć $f'(x), f''(x)$ dla podanych funkcji złożonych:

(a) $f(x)=g(x^2)$; **(b)** $f(x)=g(e^x)$; **(c)** $f(x)=g(\frac{1}{x})$; **(d)** $f(x)=g(\ln x)$; **(e)** $f(x)=g(g(x^2))$;
(f) $f(x)=e^{g(x)}$; **(g)** $f(x)=xg(3x)$;

Zad 6. Zakładając, że funkcje $f(x)$ i $g(x)$ posiadają pochodne właściwe, obliczyć pochodne funkcji:

(a) $y(x)=\log_{f(x)} g(x)$; **(b)** $y(x)=\sin\frac{f(x)}{g(x)}$; **(c)** $y(x)=\sqrt[3]{f^2(x)+g^2(x)}$; **(d)** $y(x)=\frac{\sin f(x)}{\cos g(x)}$;

Zad 7. Wyprowadzić wzór na n-tą pochodną funkcji:

(a) $f(x)=\sin x$; **(b)** $f(x)=\cos(-2x)$; **(c)** $f(x)=e^{-3x}$; **(d)** $f(x)=e^x\sin x$; **(e)** $f(x)=xe^x$;
(f) $f(x)=\ln(1-x)$; **(g)** $f(x)=\frac{1}{(1-x)^2}$; **(h)** $f(x)=x\ln x$;

Pochodne - odpowiedzi

Zad 1.

(a) $\frac{1}{3}x^{-\frac{2}{3}}$; **(b)** $4x - \frac{1}{2\sqrt{x^3}} + \frac{1}{x^2}$; **(c)** $-\frac{4}{3}x^{-\frac{5}{3}} - \frac{1}{3}x^{-\frac{2}{3}}$; **(d)** $7\sin x + 7x\cos x$; **(e)** $\frac{\arctan x}{2\sqrt{x}} + \frac{\sqrt{x}}{x^2+1}$;
(f) $3x^2\ln x + x^2$; **(g)** $x\sin x\log x + \frac{1}{2}x^2\cos x\log x + \frac{1}{2\ln 10}x\sin x$; **(h)** $-\frac{1}{\log_2^2 x}\frac{1}{x\ln 2}$; **(i)** $\frac{1}{\cos x+1}$;
(j) $-\frac{x^2-74x-7}{(x^2+7)^2}$; **(k)** $-e^{-x}(\sin x + \cos x)$; **(l)** $-\frac{2x^2}{(x^2+1)^2}$; **(m)** $\frac{1}{3x^{\frac{4}{3}}-6x+3x^{\frac{2}{3}}}$; **(n)** $-\frac{\arccos x}{x^2} - \frac{1}{x\sqrt{1-x^2}}$;
(o) $\frac{e^{-x}10^x\ln 10}{\sin x} - \frac{e^{-x}10^x}{\sin x} - \frac{e^{-x}10^x\cos x}{\sin x^2}$; **(p)** $\frac{4^x(\ln(4)\sin x+\cos x)}{2x^2\operatorname{tg}x+1} - \frac{4^x\sin x\left(4x\operatorname{tg}x+2x^2\sec^2 x\right)}{(2x^2\operatorname{tg}x+1)^2}$;

Zad 2.

(a) $12x\left(x^2+1\right)^5$; **(b)** $3(2x+3)\left(x^2+3x\right)^2$; **(c)** $-2\sin(2x)$; **(d)** $7\cos(7x+1)$; **(e)** $\frac{1}{3\cos^2\left(\frac{x}{3}\right)}$;
(f) $\frac{x}{\sqrt{x^2+1}}$; **(g)** $\frac{4x+\cos(x)}{2\sqrt{2x^2+\sin(x)}}$; **(h)** $\frac{1}{\sqrt{1-x^2}(1-x^2)}$; **(i)** $\frac{2(2x+1)\left(x^2+x-2\right)}{3(x^2+x-2)^{4/3}}$; **(j)** $\frac{4\tan^3(x)}{\cos^2(x)}$;
(k) $-\sin(2x)$; **(l)** $\frac{1}{(2\cos(x)+2)\sqrt{\tan\left(\frac{x}{2}\right)}}$; **(m)** $\frac{1}{\sqrt{-x^2+x+\frac{1}{2}}}$; **(n)** $\frac{1}{2x\sqrt{\ln x}}$; **(o)** $\frac{x\cos\left(\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}$;
(p) $\cos(x)\cos(\sin(x))$; **(q)** $\frac{x^2-1}{x^2\left(\cos\left(2x+\frac{2}{x}\right)+1\right)\sqrt{\tan\left(x+\frac{1}{x}\right)+2}}$; **(r)** $-\frac{\sin\left(2-\frac{4}{\sqrt{x}+1}\right)}{\left(\sqrt{x}+1\right)^2\sqrt{x}}$; **(s)** $\frac{e^{\sqrt{\ln x}}}{2x\sqrt{\ln x}}$;
(t) $2^{3^x}3^x\ln 2\ln 3$; **(u)** $e^{x^2+3x+2}(2x+3)\cos\left(e^{x^2+3x+2}\right)$; **(v)** $8\cot(8x)$; **(w)** $\frac{x^3\log^3\left(x^4+1\right)}{x^4+1}$;
(x) $\frac{e^{\frac{x}{\ln x}}(\ln x-1)}{\ln^2 x}$; **(y)** $\frac{4\left(x^5+x^2\right)\cos\left(x^2+1\right)-\left(x^3-2\right)\sin\left(x^2+1\right)}{2(x^3+1)^{3/2}}$; **(z)** $\frac{1}{x\ln(2)\ln(x)\ln(\log_5 x)}$;

Zad 3.

(a) $x^x(\ln x+1)$; **(b)** $x^{x^2}(2x\ln x + x)$; **(c)** $-\sin x^{\cos x-1}\left(\sin^2 x\ln(\sin x) - \cos^2 x\right)$;
(d) $2x^{\ln x-1}\ln x$; **(e)** $(x+1)^{\frac{2}{x}}\left(\frac{2}{x(x+1)} - \frac{2\ln(x+1)}{x^2}\right)$; **(f)** $-x^{\frac{1}{x}-2}(\ln x - 1)$;
(g) $\frac{x^{\sqrt{x}-\frac{1}{2}}(\ln x+2)}{2}$; **(h)** $x^{e^x-1}e^x(x\ln x+1)$; **(i)** $\frac{\left(1+\frac{1}{x}\right)^x\left(x\ln\left(1+\frac{1}{x}\right)+\ln\left(1+\frac{1}{x}\right)-1\right)}{x+1}$;
(j) $\ln x^{x-1}(\ln x\ln(\ln x)+1)$; **(k)** $x^{x^x}\left(x^x\ln x(\ln x+1)+x^{x-1}\right)$; **(l)** $\frac{e^x(\ln x)^{e^x-1}(x\ln x\ln(\ln(x))+1)}{x}$;
(m) $(\tan(2x))^{\cot\left(\frac{x}{2}\right)}\left(\frac{2\cot\left(\frac{x}{2}\right)}{\sin(2x)\cos(2x)} - \frac{\ln(\tan(2x))}{2\sin^2\left(\frac{x}{2}\right)}\right)$;

Zad 4.

(a) $f'(x) = \frac{2}{x^2} + 3x^2,\ f''(x) = -\frac{4}{x^3} + 6x,\ f'''(x) = 6 + \frac{12}{x^4}$;
(b) $f'(x) = x\cos x + \sin x,\ f''(x) = 2\cos x - x\sin x,\ f'''(x) = -x\cos x - 3\sin x$;
(c) $f'(x) = \frac{e^x(-1+x)}{x^2},\ f''(x) = \frac{e^x\left(2-2x+x^2\right)}{x^3},\ y'''(x) = \frac{e^x\left(-6+6x-3x^2+x^3\right)}{x^4}$;
(d) $f'(x) = x^3(1+4\ln x),\ f''(x) = x^2(7+12\ln x),\ f'''(x) = 2x(13+12\ln x)$;
(e) $f'(x) = -e^{\cos x}\sin x,\ y''(x) = e^{\cos x}\left(-\cos x + \sin^2 x\right),\ f'''(x) = \frac{1}{2}e^{\cos x}(1+6\cos x+\cos(2x))\sin x$;

Zad 5.

(a) $f'(x) = 2xg'\left(x^2\right),\ f''(x) = 2g'\left(x^2\right) + 4x^2g''\left(x^2\right)$; **(b)** $f'(x) = e^xg'\left(e^x\right),\ f''(x) = e^xg'\left(e^x\right) + e^{2x}g''\left(e^x\right)$;
(c) $f'(x) = -\frac{g'\left(\frac{1}{x}\right)}{x^2},\ f''(x) = \frac{2g'\left(\frac{1}{x}\right)}{x^3} + \frac{g''\left(\frac{1}{x}\right)}{x^4}$; **(d)** $f'(x) = \frac{g'(\ln x)}{x},\ f''(x) = \frac{-g'(\ln x)+g''(\ln x)}{x^2}$;
(e) $f'(x) = 2xg'\left(x^2\right)g'\left(g\left(x^2\right)\right),\ f''(x) = 2g'\left(x^2\right)g'\left(g\left(x^2\right)\right) + 4x^2g'\left(g\left(x^2\right)\right)g''\left(x^2\right) + 4x^2\left(g'\left(x^2\right)\right)^2g''\left(g\left(x^2\right)\right)$;
(f) $f'(x) = e^{g(x)}g'(x),\ f''(x) = e^{g(x)}\left(g'(x)\right)^2 + e^{g(x)}g''(x)$; **(g)** $y'(x) = g(3x) + 3xg'(3x),\ f''(x) = 6g'(3x) + 9xg''(3x)$;

Zad 6.

(a) $y'(x) = -\frac{\ln g(x)f'(x)}{f(x)\ln^2 f(x)} + \frac{g'(x)}{g(x)\ln f(x)}$; **(b)** $y'(x) = \cos\left(\frac{f(x)}{g(x)}\right)\left(\frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{g^2(x)}\right)$;
(c) $y'(x) = \frac{2f(x)f'(x)+2g(x)g'(x)}{3(f^2(x)+g^2(x))^{2/3}}$; **(d)** $y'(x) = \frac{\cos(f(x))}{\cos(g(x))}f'(x) + \frac{\sin(f(x))}{\cos(g(x))}\tan(g(x))g'(x)$;

Zad 7.

(a) $f^{(n)}(x) = \sin\left(x + n\frac{\pi}{2}\right)$; **(b)** ; **(c)** $f^{(n)}(x) = (-3)^n e^{-3x}$; **(d)** ; **(e)** $f^{(n)}(x) = (n+x)e^x$;
(f) $f^{(n)}(x) = \frac{-(n-1)!}{(1-x)^n}$; **(g)** $f^{(n)}(x) = \frac{-(n+1)!}{(1-x)^{n-2}}$; **(h)** ;